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WEAK LOGICS AND THEIR MODELS

Yuich Komori

There are many nonclassical logics, for example, intuitionistic logic, Lukasiewicz's many valued logics, intermediate logics, quantum logics, relevant logics and so on. In studying Lukasiewicz logics (cf. (2)), we noticed that the logic, obtained by deleting the contraction rule from Gentzen's LJ, is weaker than \( L_0 \)-valued Lukasiewicz logic. Of course, the logic is weaker than intuitionistic logic. Afterwards we found out the close relation between the logic and Iseki's BCK-algebra. So, we have named the logic \( L_{BCK} \). As Gentzen-type formulation is a useful tool in studying BCK-algebras, progress in it has been made since then (cf. (1),(5)). We have found out Kripke type semantics for \( L_{BCK} \), defined by using structures which are monoids and meet-semilattices at the same time (cf. (4)). It should be mentioned that we have gotten a hint of our semantics from Idziak's work (1). Moreover, in the same way, we have gotten Kripke semantics for the logic (named \( L_{BCA} \)) obtained by deleting all structure rules from LJ (cf. (3),(4)). The logic \( L_{BCA} \) is a fundamental logic, that is, we can obtain almost all of the nonclassical logics by adding some axioms and rules of inference to \( L_{BCA} \). So, it turned out that we can deal uniformly with these logics, which have been studied separately until now. At the end, we illustrate the inclusion relationships between important logics stronger than \( L_{BCA} \). Here, for example, \( L_{BCC} \) means the logic obtained from \( L_{BCA} \) adding weakening rule.
c: contraction rule, w: weakening rule, ex: exchange rule, d: distributive law \[ \alpha \land (\beta \lor \gamma) \rightarrow (\alpha \land \beta) \lor (\alpha \land \gamma). \]

REFERENCES

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Department of Mathematics
Faculty of Science
Shizuoka University
Ohya, Shizuoka, JAPAN