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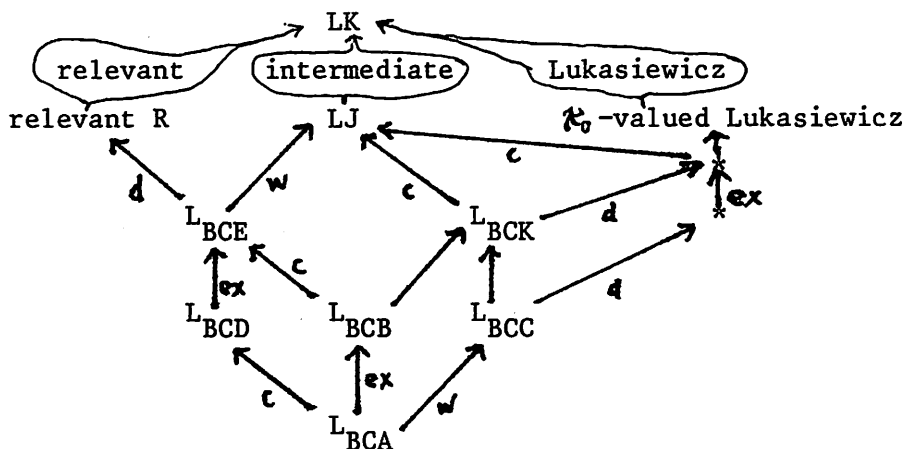——Semigroup Theory and its Related Fields——

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WEAK LOGICS AND THEIR MODELS

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There are many nonclassical logics, for example, intuitionistic
logic, Lukasiewicz's many valued logics, intermediate logics,
quantum logics, relevant logics and so on. In studying Lukasiewicz
logics (cf. (2)), we noticed that the logic, obtained by
deleting the contraction rule from Gentzen's LJ, is weaker
than $\aleph_0$-valued Lukasiewicz logic. Of course, the logic is
weaker than intuitionistic logic. Afterwards we found out
the close relation between the logic and Iseki's BCK-algebebra.
So, we have named the logic L_{BCK}. As Gentzen-type formulation
is a useful tool in studying BCK-algebras, progress in it has
been made since then (cf. (1),(5)). We have found out Kripke
type semantics for L_{BCK}, defined by using structures which
are monoids and meet-semilattices at the same time (cf. (4)).
It should be mentioned that we have gotten a hint of our semantics
from Idziak's work (1). Moreover, in the same way, we have
gotten Kripke semantics for the logic (named L_{BCA}) obtained
by deleting all structure rules from LJ (cf. (3),(4)). The
logic L_{BCA} is a fundamental logic, that is, we can obtain almost
all of the nonclassical logics by adding some axioms and rules
of inference to L_{BCA}. So, it turned out that we can deal uniformly
with these logics, which have been studied separately until
now. At the end, we illustrate the inclusion relationships
between important logics stronger than L_{BCA}. Here, for example,
L_{BCC} means the logic obtained from L_{BCA} adding weakening rule.

LK

relevant intermediate Lukasiewicz

relevant R   LJ $\aleph_0$-valued Lukasiewicz

d w c c d ex

L_{BCE} L_{BCK}

ex c d

L_{BCD} L_{BCB} L_{BCC}

c ex w

L_{BCA}

c: contraction rule,   w: weakening rule,   ex: exchange rule,

d: distributive law $\alpha \wedge (\beta \vee \gamma) \to (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$.

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