Predicate Logics without the Structure Rules

(From Introduction of [6])

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Abstract. In our previous paper [5], we have studied Kripke-type semantics for propositional logics without the contraction rule. In this paper, we will extend our argument to predicate logics without the structure rules. Similarly to the propositional case, we can not carry out Henkin's construction in the predicate case. Besides, there exists a difficulty that the rules of inference $(\rightarrow\forall)$ and $(\exists\rightarrow)$ are not always valid in our semantics. So, we have to introduce a notion of normal models. Recently, R. B. White has proved in [7] that a comprehension principle $\exists y\forall x(x\in y \leftrightarrow A(x,y))$ can be consistently added to the predicate logic LBCK. Our semantics together with White's consistency result will give a model of the set theory based on LBCK.

In this paper, we will introduce five predicate logics without the some structure rules (except LJ), LBCA, LBCB, LBCC, LBCK, and LJ. For each of them, the cut elimination theorem holds. Then, we will introduce a Kripke type semantics with varying domain (a semantics with constant domain has been given by H. Ono [4]). In the proofs of the completeness theorems for those logics (except LJ), we can

not construct Henkin's theory. It is closely related with the fact that neither the sequent $A\wedge(B\vee C)\rightarrow(A\wedge B)\vee(A\wedge C)$ nor the sequent $A\wedge\exists xB(x)\rightarrow\exists x(A\wedge B(x))$ can be proved in those logics without the contraction rule. So, we must prove the completeness theorems for those logics without using Henkin's construction by changing the interpretation of $\vee$ and $\exists$. We assume the familiarity with [3] and [5].

References

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