情報論理学II 試験 3

2003 年 1 月 31 日 (金)

1. CL 項　$\lambda^* xy.xyy$　を求めよ (S, K で表せ)。

2. CL 項　$(\lambda^* xy.xyy)xy$　を weak reduction せよ。

3. CL 項　$\lambda^* xy.xyy$　が持つ型 δ を求め，　$\lambda^* xy.xyy \in \delta$　に至る TA_C の証明図をかけ。

4. ラムダ項　$\lambda xy.(\lambda uz.y(xz))(\lambda y.yz)$　が持つ型 δ を求め,
   $\lambda xy.(\lambda uz.y(xz))(\lambda y.yz) \in \delta$　に至る TA_λ の証明図をかけ。

5. ラムダ項　$\lambda xy.(\lambda uz.y(xz))(\lambda y.yz)$　の β 正規形 (β-nf) M を求め, 4 で求めた δ に対して　$M \in \delta$　に至る TA_λ の証明図をかけ。

定義 0.1 (Weak reduction) Any CL-term KXY or $SXYZ$ is called a (weak) redex. Contracting an occurence of a redex in a term U means replacing one occurence of

$$\begin{array}{lll} KXY & \text{by} & X, \\ SXYZ & \text{by} & XZ(YZ). \end{array}$$

If this changes U to U', we say that U (weakly) contracts to U' , or

$$U \rhd_{1w} U'.$$

We say that U (weakly) reduces to V, or

$$U \rhd_w V$$

iff V is obtained from U by a finite (perhaps empty) series of weak contractions.

定義 0.2 (Abstraction) For each CL-term M and each variable x, a CL-term $\lambda^* x.M$ is defined by induction on M, thus:

$$\begin{array}{lllll} (a) & \lambda^* x.M & \equiv & KM & \text{if } x \notin FV(M); \\ (b) & \lambda^* x.x & \equiv & SKK & ; \\ (c) & \lambda^* x.Ux & \equiv & U & \text{if } x \notin FV(U); \\ (d) & \lambda^* x.UV & \equiv & S(\lambda^* x.U)(\lambda^* x.V) & \text{if neither (a) nor (c) applies.} \end{array}$$

定義 0.3 (The type-assignment system $\mathbf{TA}_C$) It has two axiom-schemes; they are

$$\begin{array}{ll} (\to K) & K \in \alpha \to \beta \to \alpha, \\ (\to S) & S \in (\alpha \to \beta \to \gamma) \to (\alpha \to \beta) \to \alpha \to \gamma. \end{array}$$

Its only rule is called the →-elimination rule or (→e);

$$\frac{X \in \alpha \to \beta \quad Y \in \alpha}{XY \in \beta} (\to \text{e}).$$

定義 0.4 (The type-assignment system $\mathbf{TA}_\lambda$) It has no axiom-schemes and two rules the →-elimination rule (→e) and the →-introduction rule (→i);

$$\frac{X \in \alpha \to \beta \quad Y \in \alpha}{XY \in \beta} (\to \text{e}),$$

$$\frac{\begin{array}{c}[x \in \alpha] \\ M \in \beta\end{array}}{\lambda x.M \in \alpha \to \beta} (\to \text{i}).$$